 LSU Math Circle Research Proposal

Completeness of Distributions

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Category: Probability Theory (Calculus not required)

Abstract: In the space of probability distributions on a countable space, such as the integers, we can easily define metrics within the space. The set of probability distributions is a subset of the infinite-dimensional space $\mathbb{R}^\infty$, and while the space it resides in can be made into a vector space under pointwise addition, the set of probability distributions is easily seen to not be a subspace. However, we can still discuss the “closeness” of vectors residing in the set of distributions. The concept of completeness, in a metric space, is something often studied by those getting into introductory analysis. Being that the set of probability distributions on the integers can be made into a metric space, we will attempt to determine whether the set is complete under a given metric. That is, for a Cauchy sequence of distributions $a_n$ in the set of distributions equipped with the metric $\rho$, does the property

$$\lim_{m,n \to \infty} \rho(a_n, a_m) \to 0$$

imply that

$$\lim_{n \to \infty} a_n \to a$$

for a distribution $a$ in the set of probability distributions? We will start with the supremum metric. That is the metric $\rho$, defined on vectors $a$ and $b$ with

$$\rho(a, b) = \sup_k |a_k - b_k|$$

where $a_k$ and $b_k$ are the entries in the $k^{th}$ coordinate of the vectors $a$ and $b$ respectively. Once a more thorough understanding of metrics is gained, we will determine what is not a metric on the set, and examine the completeness of other metrics on the set. Introductory topics in linear algebra, such as vector spaces, will be discussed as well.