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Category: Probability Theory (Calculus not required)

Abstract: In the space of probability distributions on a countable space, such as the integers, we can easily define metrics within the space. The set of probability distributions is a subset of the infinite-dimensional space $\mathbb{R}^\infty$, and while the space it resides in can be made into a vector space under pointwise addition, the set of probability distributions is easily seen to not be a subspace. However, we can still discuss the “closeness” of vectors residing in the set of distributions.

The concept of completeness, in a metric space, is something often studied by those getting into introductory analysis. Being that the set of probability distributions on the integers can be made into a metric space, we will attempt to determine whether the set is complete under a given metric. That is, for a Cauchy sequence of distributions $a_n$ in the set of distributions equipped with the metric $\rho$, does the property

$$\lim_{m,n \to \infty} \rho(a_n, a_m) \to 0$$

imply that

$$\lim_{n \to \infty} a_n \to a$$

for a distribution $a$ in the set of probability distributions? We will start with the supremum metric. That is, the metric $\rho$, defined on vectors $a$ and $b$ with

$$\rho(a,b) = \sup_k |a_k - b_k|$$

where $a_k$ and $b_k$ are the entries in the $k^{th}$ coordinate of the vectors $a$ and $b$ respectively. Once a more thorough understanding of metrics is gained, we will determine what is not a metric on the set, and examine the completeness of other metrics on the set. Introductory topics in linear algebra, such as vector spaces, will be discussed as well.

This research proposal has the potential for continued research after the program.

REFERENCES