LSU Math Circle Research Proposal

Unavoidable Induced Subgraphs of Large Rooted Trees

Instructor: Samuel Weiner
Session: June 13 - July 2, 2022
Category: Combinatorics **(Calculus not required)**

Abstract: In the field of graph theory, a graph is a collection of vertices and edges. Graphs are powerful tools that can be used to model any real-world system. For instance, consider air traffic: each airport can be viewed as a vertex, and two vertices are connected by an edge if there is a flight that travels between the corresponding airports. Since there are so many flights from so many airlines at so many different times of day, it can be difficult to find the cheapest or quickest travel option. This is an issue across any large or complex system: how do we sift through the noise to find what we need? This project seeks to answer this question by using an active area of research within graph theory known as Ramsey Theory, which helps us find the basic structural properties common to even the most complex systems in existence.

In the aforementioned graph modeling air traffic, let us imagine that we take some large collection of edges and color them red. If we can find all subgraphs which contain many of these red edges, then we would know exactly how some cities are connected through air travel. We could then use this discovery to know with certainty the cheapest or quickest ways to travel between any two given cities. As such, the crux of the issue is to know how to find the subgraphs that contain these red edges, and that is what this research project seeks to do.

Project Outline:

Week 1 (Introduction to Graph Theory and Project Goals)
The first week will be spent learning basic graph theory, with a focus on topics that are foundational to Ramsey Theory. These include but are not limited to *connectivity, subgraphs and induced subgraphs,* and *Ramsey's Theorem*. Students will be taught from the instructor's notes, which are based off [1]. Students need not purchase any textbooks for this session but are welcome to use [1] as a reference and for extra practice if they so choose. This period of instruction will also include an introduction to proof writing, emphasizing methods most commonly used in combinatorics such as *proof by induction* and *proof by contradiction*. Students will be exposed to practice exercises to bolster their understanding of these concepts, both in class and for homework.

Towards the end of the week, some contemporary preliminary results relating to this research, such as [2], will also be introduced and recontextualized in relation to the project goals.

Week 2 (Preliminary Experimentation and Research)
The second week is where the bulk of the research takes place. To start, the students will begin by applying the research hypothesis to small graph examples to get comfortable with the methods used and the rough idea of what their results might look like. From there, we will move

into more supervised research for the general case. The goal for the end of the week is to have a finalized list of subgraphs for the result.

At the end of the week, students will be given tips and guidelines for the write-up of the main result and will be asked to start brainstorming the structure and proof methods of their argument.

Week 3 (Formal Write-Up of the Result and Presentation)
After arriving at the result, the students will begin to work on writing a rigorous proof of the main result, utilizing, and building upon their ideas from the aforementioned brainstorming session at the end of Week 2. Once that is completed, students will spend at least two class sessions writing the slides and practicing their presentation. Students will be encouraged to use beamer for the final presentation (with a template provided by the instructor), but the decision will ultimately rest with them.

Skills and Background Knowledge: Interested students need only be familiar with high school algebra and basic arithmetic. Some experience with proof-based mathematics will be helpful but is not required. All other skills will be taught by the instructor, including enumerative combinatorics, mathematical proofs, Ramsey Theory, and others.

References

[1] R. Diestel, *Graph Theory*, 5th ed., Graduate Texts in Mathematics, 2016.
[2] Bogdan Oporowski, James Oxley, and Robin Thomas, Typical Subgraphs of 3- and 4-connected graphs, *Journal of Combinatorial Theory*, Series B57 (1993) 239-257.